

Simplifying Rational Expressions

Some rules used when simplifying rational expressions, for $b \neq 0$, $c \neq 0$

$$1.) \frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$$

$$2.) \frac{ac}{bc} = \frac{a}{b}$$

To Simplify a Rational Expression

1.) Completely factor the numerator (top) and the denominator (bottom).

a. If you can't find factors, you can try polynomial long division.

2.) Cancel common factors, do not cancel terms only factors! Remember, $\frac{a}{a} = 1$.

$$\frac{a^2 + a}{a} = \frac{a^2 + \cancel{a}}{\cancel{a}} = a^2$$

a. Wrong:

$$\frac{a^2 + a}{a} = \frac{a(a+1)}{a} = \frac{\cancel{a}(a+1)}{\cancel{a}} = a+1$$

b. Right:

3.) Note the values for which this is not true. These are the "bad values of x", the values that cause the denominator to be 0.

a. Above, b is true when $a \neq 0$.

Example 1:

$$\frac{t^2 - 1}{t^2 + 7t + 6} \xrightarrow[\substack{\text{Difference of Squares} \\ 6+1=7, \text{ same signs}}]{\text{}} \frac{(t-1)(t+1)}{(t+1)(t+6)} \xrightarrow{\text{cancel}(t+1)} \frac{t-1}{t+6}, \text{ so}$$

long as $t \neq -1$ & $t \neq -6$.

Example 2:

$$\frac{7 - 34x - 5x^2}{25x^2 - 1} \xrightarrow[\text{factor}(-1)\text{ on top}]{\text{}} \frac{(-1)(5x^2 + 34x - 7)}{25x^2 - 1} \xrightarrow[\text{Difference of Squares}]{\text{use } 5*7-1} \frac{(-1)(5x-1)(x+7)}{(5x-1)(5x+1)} \xrightarrow{\text{cancel}(5x-1)} \frac{(-1)(x+7)}{(5x+1)} \text{ or } \frac{-x-7}{5x+1} \text{ or } \frac{x+7}{-5x-1} \text{ or } -\frac{x+7}{5x+1}$$

provided $x \neq \pm \frac{1}{5}$.

Example 3:

$$\frac{y-9}{9-y} \xrightarrow{\text{Rearrange to same order}} \frac{y-9}{-y+9} \xrightarrow{\text{Factor out } -1} \frac{y-9}{-1*y - 1*9} \rightarrow \frac{(y-9)}{-1(y-9)} \xrightarrow{\text{Cancel}(y-9)} \frac{1}{-1} \rightarrow -1, \text{ Provided } y \neq 9$$