Applications for Rational Equations

Problem #1 - Work Rate
Machine 1 can do a job in 5 hours. Machine 2 can do the job in 3 hours. How much time is required to complete the job if the two machines work together?

Keep in mind that 2 machines working together should take less time than the fastest on own.

Method 1: In one unit of time… (1 hour)

rate per hour machine 1 works is 1/5 of the job per hour
rate per hour machine 2 works is 1/3 of the job per hour

Let x = time required if the machines work together. So the rate of the two machines working together is 1/x of the job per hour.

part done by machine 1 + part done by machine 2 = part they do together

$\frac{1}{5}+\frac{1}{3}=\frac{1}{x}$
LCD = 15x

$$15x\left(\frac{1}{5}+\frac{1}{3}\right)=15x\frac{1}{x}\xrightarrow{Distribute 15x}15x\frac{1}{5}+15x\frac{1}{3}=15x\frac{1}{x}\xrightarrow{Cancel}3x+5x=15$$

$$\xrightarrow{CombineLikeTerms}8x=15\xrightarrow{DivideBothSidesBy8}x=\frac{15}{8}$$

So if the machines work together they will take 7.5 hours to do the job.

Method 2: **rate** times **time** for **each machine** added together equals **the whole job**
$\frac{1}{5}x+\frac{1}{3}x=1$ → 1 means the whole job
LCD = 15, multiplying both sides by 15, we get 3x + 5x = 15 & so x = 7.5

Problem 2 - Work Rate

John can paint a room in 3 hours. Sue can paint the same room in 2.5 hours. How much time is required to paint the room if they work together?

Let x = time required working together

Method 2: **rate** times **time** for each added together equals **the whole job**
With this method we could chart the problem like we do the distance problems:

	rate	time worked together	Part of job completed
John	$\frac{1}{3}$	x	$\frac{1}{3}x$
Sue	$\frac{1}{2.5}$	x	$\frac{1}{2.5}x$
Together			1

Adding the part of job completed column we get:
$\frac{1}{3}x+\frac{1}{2.5}x=1$, a common denominator is (3)(2.5), use this product to cancel denominators as above.

$$(3)(2.5)\left(\frac{1}{3}x+\frac{1}{2.5}x\right)=(3)(2.5)1 \rightarrow (\cancel{3})(2.5)\frac{1}{\cancel{3}}x+(3)(\cancel{2.5})\frac{1}{\cancel{2.5}}x=7.5$$

$$\rightarrow 2.5x+3x=7.5 \rightarrow 5.5x=7.5 \rightarrow x=1\frac{4}{11}$$

Proportions & Problem Solving

A ratio is the quotient of two numbers or quantities, 3 problems out of 5 problems & can be written as $\frac{3\text{ problems}}{5\text{ problems}} = \frac{3}{5}$. Rate is a type of ratio comparing different units, 15 miles in 1 hour = $\frac{15\text{ miles}}{1\text{ hour}}$ = 15 mph. When we have a ratio on each side of the equals sign it is called a proportion, 1 inch is to 12 inches as 3 inches are to 36 inches → $\frac{1\text{ inch}}{12\text{ inches}} = \frac{3\text{ inches}}{36\text{ inches}}$. Often we leave out the units when we are working on the problem, but it is necessary to make sure they are correct all through the problem. To solve a proportion we can use the "cross-multiply" method.

Problem 3 - Uniform Motion use d=rt:

A freight train travels 100 miles in the same time that an express train travels 150 miles. If the express train goes 20 mph faster than the freight train, find the rate of each.

	d	r	t
freight	100	r	
express	150	r + 20	

The concept is freight time = express time, and time equals distance divided by rate or $t = \frac{d}{r}$.

$$\frac{100}{r} = \frac{150}{r+20}$$

Since this is a proportion, fraction = fraction, we can use the "cross-multiply" method to solve, $\frac{100}{r} = \frac{150}{r+20}$

100(r+20) = 150r → 100r + 2000 = 150r → 2000 = 50r → 40 = r.
The rate of the freight train is 40 mph.
The rate of the express train is rate of freight + 20, so the rate of the express is 60 mph.

Problem 4 - Uniform Motion use d=rt:

A small plane has a speed of 170 mph in still air. Find the speed of the wind if the plane travels a distance of 400 miles with a tail wind in the same time it takes to travel 280 miles into a head wind.

Let x = wind speed

	d	r	t
with wind	400	170 + x	
against wind	280	170 – x	

Again, the plane is taking the same amount of time to go 2 distances, so we use d/r = t.

$\frac{400}{170+x} = \frac{280}{170-x}$, this is again a proportion, $$\frac{400}{170+x} = \frac{280}{170-x}$$

400(170 – x) = 280(170 + x) → 68000 – 400x = 47600 + 280x → 20400 = 680x → 30 = x

The speed of the wind is 30 mph.

Problem 5 – Different Distances and Different Times, but Related Rates, use d=rt: Ekatrina has to go to a meeting for her job. She travels 1080 miles by jet, and then she takes a rental car 240 miles. Ignoring time wasted not in a vehicle, she travels for one hour more in the car than she does in the jet. The jet travels at six times faster than her average speed in the car. Find the time she spends in the jet, and the time she spends in the car.

Since we are asked to find time, leave time as the unknown.

Let t = time in the jet. We are given that time in the car is 1 hour more than the time in the jet, so time in the car = t + 1.

Since d = rt, we can solve for r, so $r = \frac{d}{t}$. We know the distance traveled by each vehicle, and the time spent in each vehicle. So we have the rate for each vehicle.

The rate of the jet = $\frac{1080}{t}$.

The rate of the car = $\frac{240}{t+1}$.

Now we are also told that the rate of the jet is six time faster than the rate of the car.

$$\frac{1080}{t} = 6\left(\frac{240}{t+1}\right)$$

We now have one equation in one unknown so we can solve this problem.

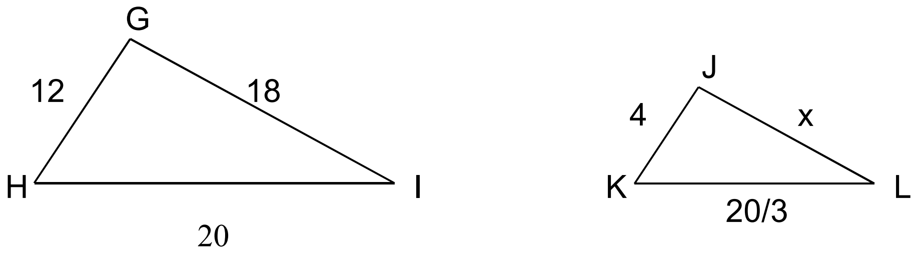
$\frac{1080}{t} = 6\left(\frac{240}{t+1}\right)$ My recommendation is to divide both sides by 6, so we have a proportion. Remember, dividing by 6 is the same as multiplying by $\frac{1}{6}$.

$\frac{1080}{6t} = \frac{240}{t+1}$	Now we can "Cross-Multiply"
1080(t + 1) = 240(6t)	Distribute.
1080t + 1080 = 1440t	Subtract 1080t from each side.
1080 = 360t	Divide both sides by 360.
3 = t	

So Ekatrina spent 3 hours in the jet. She spent 1 more hour in the car, so she spent 4 hours traveling by car.

Problem 6 - Similar Triangles

Similar Triangles angles are equal, sides are proportional.



$\frac{12}{4} = \frac{18}{x}$, is one of several proportions we could set up.

12x = 72 → x = 6

Harmonic Mean

As its name implies the Harmonic Mean was used in Ancient Greece for calculating the ratio of the string length to create “harmonious music”. There are various sites on the internet that go into how the formula can be derived. However, we will just use the formula for an example of Rational Equation Applications.

Harmonic Mean = $\dfrac{n}{\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3}+\cdots+\frac{1}{x_n}}$, where n is the number of values.

Find the Harmonic Mean of: 1.32, 1.40, 1.57, 1.68, 1.62, 1.78

There are 6 values, so n = 6

$$\dfrac{6}{\frac{1}{1.32}+\frac{1}{1.40}+\frac{1}{1.57}+\frac{1}{1.68}+\frac{1}{1.62}+\frac{1}{1.78}} \xrightarrow{Use\_Product\_of\_Denominators}$$

$$\dfrac{6}{\dfrac{7.278+6.862+6.119+5.7184+5.93+5.397}{9.60696}} \to \dfrac{6}{\dfrac{37.3044}{9.60696}} \to \dfrac{6}{3.883} \to 1.545$$

Rounded to 3 significant digits the answer is 1.55, since that is how many we have in the problem. Don’t round until done…

Now, suppose you want to include a seventh value to make the Harmonic Mean equal to 1.58. The set up would be:

$$1.58 = \frac{7}{\frac{1}{1.32}+\frac{1}{1.40}+\frac{1}{1.57}+\frac{1}{1.68}+\frac{1}{1.62}+\frac{1}{1.78}+\frac{1}{x_7}}$$

You now need to solve for x_7.

First "Cross-Multiply", remember the denominator of 1.58 is 1.

$$\left(\frac{1}{1.32}+\frac{1}{1.40}+\frac{1}{1.57}+\frac{1}{1.68}+\frac{1}{1.62}+\frac{1}{1.78}+\frac{1}{x_7}\right)1.58 = 7(1)$$

Now, inside the ( ) you have the variable term, its only coefficient is 1.58, so divide both sides by 1.58.

$$\frac{1}{1.32}+\frac{1}{1.40}+\frac{1}{1.57}+\frac{1}{1.68}+\frac{1}{1.62}+\frac{1}{1.78}+\frac{1}{x_7} = \frac{7}{1.58}$$

Now, the only term with a variable is $\frac{1}{x_7}$, so subtract everything else from both sides to get that term by itself.

$$\frac{1}{x_7} = \frac{7}{1.58} - \left(\frac{1}{1.32}+\frac{1}{1.40}+\frac{1}{1.57}+\frac{1}{1.68}+\frac{1}{1.62}+\frac{1}{1.78}\right)$$

Since the x_7 is in the denominator, we can "Cross-Multiply" again, but remember EVERYTHING on the right will go to the denominator.

$$\frac{1}{\frac{7}{1.58} - \left(\frac{1}{1.32}+\frac{1}{1.40}+\frac{1}{1.57}+\frac{1}{1.68}+\frac{1}{1.62}+\frac{1}{1.78}\right)} = x_7$$

When calculating, don't forget the (-) in front of the parenthesis!

So x_7 = 1.83 rounded to 2 decimal places, since we have 3 significant digits.

Some interesting properties of the Harmonic Mean, including distance for car travel, are listed on the Ask Dr. Math Website: http://mathforum.org/library/drmath/view/57565.html

Solving Formulae, Literal Equations and other equations for a given variable.

1.) Is the variable in the numerator of a fraction
    a. Yes, go to step 2
    b. No, Multiply both sides of the equation by the LCD of the fractions containing the required variable – often the easiest method is to multiply the whole equation by the product of all denominators clearing ALL fractions.
2.) Move all the terms containing the given variable to one side of the equation and everything else to the other side.
3.) Simplify each side as much as possible.
4.) Is the given variable in more than one term that cannot be combined
    a. No, go to step 5.
    b. Yes, Factor the required variable out of every term possible.
        i. If you only have a product of factors left go to step 5
        ii. If you have additional terms you may need to double check your work or start again.
5.) Divide both sides of the equation by the "coefficient" of the given variable to get it by itself.

Example

Solve for B, $T = \dfrac{2U}{B+E}$

$$(B+E)T = (B+E)\frac{2U}{B+E}$$

$$BT + ET = 2U$$

$$BT + ET - ET = 2U - ET$$

$$\frac{BT}{T} = \frac{2U - ET}{T}$$

$$B = \frac{2U - ET}{T}$$

Solve for R_2, $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$, LCD$=RR_1R_2$

$$(RR_1R_2)\frac{1}{R} = (RR_1R_2)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$R_1R_2 = (RR_1R_2)\frac{1}{R_1} + (RR_1R_2)\frac{1}{R_2}$$

$$R_1R_2 = RR_2 + RR_1$$

$$R_1R_2 - RR_2 = RR_2 - RR_2 + RR_1$$

$$(R_1 - R)R_2 = RR_1$$

$$\frac{(R_1 - R)R_2}{(R_1 - R)} = \frac{RR_1}{(R_1 - R)}$$

$$R_2 = \frac{RR_1}{(R_1 - R)}$$