Rational Equations

To Solve a Rational Equation:

1.) Determine what value(s) of the variable won't work in the equation.

   a. Factor all denominators
   b. Set each denominator equal to zero.
   c. These values will not be in the domain.

2.) Remove Denominators.

   a. Find the LCD, may be as simple as the product of factors found in part 1.)
   b. Multiply both sides of the equation by the LCD.

3.) Solve, now we have a Linear or Polynomial Equation:

   a. Remove any grouping symbols (distribute)
   b. simplify (combine like terms)
   c. solve for the variable

4.) Check your solution in the original equation

   a. Is your solution one of the values found in step 1? If it is, it is not a solution to the Rational Equation.
   b. If your solution is not one of the values found in step 1, then it is a solution to the Rational Equation.
   c. If you are working a word problem/application, does it make sense? Remember we don't normally want negative length or time or…

Example 1: $\frac{1}{x}+\frac{1}{x-3}=\frac{9}{x^2-3x}$

1.) Determine what value(s) of the variable won't work in the equation
   a. Factor all denominators: x, x – 3, x(x – 3)
   b. Set each denominator equal to zero.
      i. x = 0
      ii. x – 3 = 0 → x = 3
      iii. x(x – 3) = 0 → x = 0 & x – 3 = 0 → x = 3
   c. These values will not be in the domain. The domain is { x| x ≠ 0 & x ≠ 3}.

2.) Remove Denominators.
   a. The LCD: x(x – 3)
   b. Multiply both sides of the equation by the LCD.

$$(x(x-3))\left(\frac{1}{x}+\frac{1}{x-3}\right)=(x(x-3))\left(\frac{9}{x^2-3x}\right)$$

      i. $\xrightarrow{Distribute}(x(x-3))\frac{1}{x}+(x(x-3))\frac{1}{x-3}=(x(x-3))\left(\frac{9}{x^2-3x}\right)$

$$\xrightarrow{Cancel}(x-3)+x=9$$

3.) Solve: $(x-3)+x=9\xrightarrow{Simplify}2x-3=9\xrightarrow{+3}2x=12\xrightarrow{\div 2}x=6$

4.) Check your solution in the original equation
   a. x = 6 is in the domain, so it is a solution to the equation.

Example 2: $\frac{1}{5x}-\frac{1}{4x}+\frac{1}{3x}=-\frac{17}{60}$

1.) Determine what value(s) of the variable won't work in the equation.
   a. Factor all denominators: 5x, 4x, 3x, 20*3
   b. Set each denominator equal to zero.
      i. 5x = 0 → x = 0; 4x = 0 → x = 0; 3x = 0 → x = 0
      ii. 60 ≠ 0
   c. The value x = 0 can not be in the domain. The domain is { x| x ≠ 0}.

2.) Remove Denominators.
   a. The LCD: 60x
   b. Multiply both sides of the equation by the LCD.
      i. $$60x\left(\frac{1}{5x}-\frac{1}{4x}+\frac{1}{3x}\right)=60x\left(-\frac{17}{60}\right)\xrightarrow{Distribute}$$
      $$60x\frac{1}{5x}-60x\frac{1}{4x}+60x\frac{1}{3x}=60x\left(-\frac{17}{60}\right)\xrightarrow{Cancel}12-15+20=-17x$$

3.) Solve: $12-15+20=-17x \rightarrow 17=-17x \xrightarrow{\div -17} -1=x$
4.)
5.) Check your solution in the original equation
   a. -1 is in the domain so x = -1 is a solution.

Example 3: $\frac{x}{x+6}-3=1-\frac{6}{x+6}$

1.) Determine what value(s) of the variable won't work in the equation
   a. x + 6 = 0 → x = - 6
   b. Domain { x| x ≠ -6}

2.) Remove Denominators, LCD = x + 6
   a. $(x+6)\frac{x}{x+6}-(x+6)3=(x+6)1-(x+6)\frac{6}{x+6}\xrightarrow{Cancel}x-(x+6)3=(x+6)-6$

3.) Solve: $x-(x+6)3=(x+6)-6\xrightarrow{Distribute}x-3x-18=x+6-6\xrightarrow{Simplify}$
$-2x-18=x\xrightarrow{+2x}-18=3x\xrightarrow{\div 3}-6=x$

4.) Check your solution in the original equation
   a. -6 is not in the domain, so it is not a solution to the equation.
   b. Since there is no other solution, there is no solution to the equation.

Example 4: $\frac{15}{x+4} = \frac{x-4}{x}$

1) x = 0 & x = - 4 are not in the domain.

2) Since this is a proportion, we can use the "Cross – Multiply" Method.

    a. $\frac{15}{x+4} = \frac{x-4}{x}$ Cross – Multiply

3) Solve:

$15(x) = (x + 4)(x - 4)$		FOIL
$15x = x^2 - 16$		Set Equation equal to zero.
$0 = x^2 - 15x - 16$		Factor
$0 = (x - 16)(x + 1)$		Now set each Linear Factor equal to zero.
$x - 16 = 0$	$x + 1 = 0$	Now solve each Linear Equation.
$x = 16$	$x = - 1$	

4) Check: 16 & -1 are both in the domain.