

Special Products

For Multiplying Polynomials

Note: a & b below represent terms, not just numbers.

The **FOIL**(**F**irst **O**uter **I**nners **L**ast) Method is used when multiplying two binomials.

$$(a + b)(c + d) = ac + ad + bc + bd$$

Firsts are $a * c$

Outers are $a * d$

Inners are $b * c$

Lasts are $b * d$

F + **O** + **I** + **L** → **FOIL**

We can check this by applying the rules for Multiplying Polynomials.

$$(a + b)(c + d) \quad \text{Distribute}$$

$$a(c + d) + b(c + d) \quad \text{Distribute AGAIN!}$$

$$ac + ad + bc + bd \quad \text{If we were working with a specific problem we could now Simplify.}$$

Examples:

$$(3x + 2)(x - 1)$$

Firsts are $3x \& x \rightarrow 3x^2$

Outers are $3x \& -1 \rightarrow -3x$

Inners are $2 \& x \rightarrow 2x$

Lasts are $2 \& -1 \rightarrow -2$

Now we can just add them together

$$3x^2 + -3x + 2x - 2 \rightarrow 3x^2 + -x - 2 \text{ or } 3x^2 - x - 2$$

$$(4x - 1)(2x - 2)$$

$$8x^2 - 8x - 2x + 2$$

$$8x^2 - 10x + 2$$

F + **O** + **I** + **L**

FOIL

Squaring a Binomial = Perfect Square Trinomial(factoring section)

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

We can check both of these formulae by applying the FOIL Method from above.

$$(a + b)^2 \rightarrow (a + b)(a + b) \rightarrow a^2 + ab + ab + b^2 \rightarrow a^2 + 2ab + b^2$$

$$(a - b)^2 \rightarrow (a - b)(a - b) \rightarrow a^2 - ab - ab + (-b)^2 \rightarrow a^2 - 2ab + b^2$$

Examples:

$$(3x + 7)^2 \rightarrow (3x)^2 + 2(3x)(7) + (7)^2 \rightarrow 9x^2 + 42x + 49$$

$$(5y - 3)^2 \rightarrow (5y)^2 - 2(5y)(3) + (3)^2 \rightarrow 25y^2 - 30y + 9$$

Product of a Sum & Difference (Product of Conjugates) = Difference of Two Squares (Factoring Section)

$(a + b)(a - b) = a^2 - b^2$ $a - b$ is the *conjugate* of $a + b$, since the values are the same but the sign in between is opposite.

Again, we can check this with the FOIL Method.

$$(a + b)(a - b) \rightarrow a^2 - ab + ab - b^2 \rightarrow a^2 - b^2$$

Example:

$$(2x + 3)(2x - 3) \rightarrow (2x)^2 - (3)^2 \rightarrow 4x^2 - 9$$