Factoring Quadratic Expressions

$ax^2 + bx + c$

Leading Coefficient a = 1, $x^2 + bx + c = (x + s)(x + u)$:

1.) List factors of the last term, c.
2.) Sum each pair of factors, s & u.
3.) If the sum s + u = b, then done
4.) write the factors.

Example 1: $z^2 + 2z – 24$

1) c = 24, 24 = 1*24 = 2*12 = 3*8 = 4*6
   a. I listed as I did above so that I could be sure that I listed all the factors.
2) 1 + 24 = 25, 2 + 12 = 14, 3 + 8 = 11, 4 + 6 = 10
3) (z + 6)(z – 4)

Example 2: $x^2 + 7x + 12$

1) c = 12, 24 = 1*12 = 2*6 = 3*4
2) 1 + 12 = 13, 2 + 6 = 8, 3 + 4 = 7
3) (x + 3)(x + 4)

Example 3: $x^2 – x – 6$

1) c = 6, 6 = 1 * 6 = 2 * 3, keep in mind we need a + & – factor, because of – 6!
2) Since we want -1 for our coefficient we will subtract factors
   a. 1 – 6 = -5, 2 – 3 = -1, 3 – 2 = 1
3) (x – 3)(x + 2)

Example 4: $x^2 – x + 6$

1) c = 6, 6 = 1 * 6 = 2 * 3, keep in mind we need both + factors, because of + 6!
2) Since we want – 1 for our coefficient and our factors need to have same sign, we are done! 2 + 3 = 5 & 1 + 6 = 7 no matter which signs we use!
3) This expression is called prime, since it cannot be factored.

Tips: only work when a is positive, more involved when a ≠ 1:

1) When c is positive, its factors will have the same sign & the sum of the factors is b.
   a. If b is positive, the factors of c will both be positive
      $x^2 + bx + c = (x + \_\_)(x + \_\_)$
   b. If b is negative, the factors of c will both be negative
      $x^2 - bx + c = (x - \_\_)(x - \_\_)$
2) When c is negative, its factors will have opposite signs & the difference of the factors is b.
   a. If b is positive, the larger factor is positive.
      $x^2 + bx - c = (x + \text{larger factor})(x - \_\_)$
   b. If b is negative, the larger factor is negative.
      $x^2 - bx - c = (x - \text{larger factor})(x + \_\_)$

Leading Coefficient a ≠ 1, $ax^2 + bx + c = (rx + s)(tx + u)$

"Guess & Test" Method (some instructors only allow this method)

1.) List the factors of the first term and the factors of the last term.
2.) Make all the necessary combination of products & sums of the factors to see if any equal b, st+ ru = b.
3.) If one combination works, then done, write the factors.
   a) Keep in mind the tips above.

Example 1: $8m^2n^2 – 10mn + 3$

1.) 8 = 1*8 = 2*4; 3 = 1*3
2.) 1*1 + 8*3 = 25; 8*1 + 1*3 = 11; 2*1 + 4*3 = 14; 2*3 + 1*4 = 10
3.) (4mn – 3)(2mn – 1)

Example 2: $2x^2 + 13x – 7$

1.) 2 = 1*2; -7 = -1*7 = 1*(-7)
2.) 1*(-1) + 2*7 = 13; 2*(-1) + 1*7 = 5; 1*1 + 2*(-7) = -13; 2*1 + 1*(-7) = -5
3.) (x + 7)(2x – 1)

AC Method also called X Method or Grouping (for more details & examples see X Method)

1.) Find 2 numbers whose product is ac & sum is b
2.) Rewrite bx using the factors from 1.)
3.) Factor by grouping (using 4 term method in Factoring Basics)