Factoring Polynomials
Basics

Factoring polynomial expressions makes it easier to solve equations containing them. You need to be able to factor Integers to be able to factor Polynomials. We will mostly be looking at Trinomials, but we will also look at special instances of other polynomials.

Before Factoring Polynomials, you should review Factoring Real Numbers if you have not already done so.

The **<u>Greatest Common Factor(GCF)</u>** is the largest number (or variable expression) that divides the 2 or more terms involved, and is the product of all the common factors for a list of terms.

Example:

$27x = 3^3x = \mathbf{3x} * 3^2$
$15x^2 = \mathbf{3x} * 5x$
$3x^3 = \mathbf{3x} * x^2$
GCF = $\mathbf{3x}$

We can Factor the GCF out of an expression, in essence by “undoing” the Distributive Law. When factoring out “all of a term” be sure to write it as that term times 1, since you will still have a 1 left when you factor out the term, recall Prime Numbers.

Recall the **Distributive Law**: a(b + c) = ab + ac = (b + c)a
Generally, I like the GCF out front.

Examples:

42x – 7 → factor each term
7*6x – 7*1 → Distributive Law, factor out the GCF = 7 in each term
7(6x – 1)

7x + 21y – 7 → factor each term
7*1x + 7*3y – 7*1 → Distributive Law, factor out the GCF = 7 in each term
7(x + 3y – 1)

$x^9y^6 + x^3y^5 - x^4y^3 + x^3y^3$ → factor each term, in this case use your exponent rules look for the smallest exponent for each variable
$(x^3y^3)*(x^6y^3) + (x^3y^3)*(1*y^2) - (x^3y^3)*(x*1) + (x^3y^3)*(1)$ → Factor out the GCF = (x^3y^3) from each term
$(x^3y^3)*(x^6y^3 + 1*y^2 - x*1 + 1)$ → Remove “unnecessary” 1s
$(x^3y^3)*(x^6y^3 + y^2 - x + 1)$

$y^5 + 6y^4$ → y^4 is the GCF in this case
$y^4(y + 6)$

$12x^3 + 16x^2 – 8x$ → See if a GCF stands out, if not use Prime Factorization on each term
$4x*3x^2 + 4x*4x – 4x*2$ → Factor out the GCF = 4x
$4x(3x^2 + 4x – 2)$

- y – 3 → In this case we have GCF = -1
-1*y + -1*3 → Sometimes it is better to change subtraction to adding a negative before factoring.
-1(y + 3)

- 5 + y → Factor out -1
-1*5 + -1*-y → the negatives were hiding in the +! Remember product of 2 negatives is +
-1(5 – y)

x – 7 → Sometimes we need to factor out -1 even though we don't think we do.
-1*-x + -1*7
-1(-x + 7)

To factor by Grouping (4 terms):

1.) Group the terms into 2 groups of 2 terms.
2.) Factor out the GCF from each group.
3.) If there is a common binomial factor, factor it out.
    a. I often put this new GCF to the back of the problem, some people put it to the front, there is no difference as Multiplication in Commutative!
    b. If not, rearrange and try again…

Easy Examples (complicated later):

xs + xt + 3s + 3t → Notice the first two terms have an x in common(GCF1), & the second two terms have a 3 in common(GCF2).
x(s + t) + 3(s + t) → Now we see we have a binomial (s + t) that is common to both new terms (remember a term is a product), again we undo the Distributive Law or "Factor Out" the (s + t)
(x + 3)(s + t)

$10x^2 + 2xy – 25x – 5y$ → GCF1 = 2x, GCF2 = - 5
2x(5x + y) – 5(5x + y) → Now our GCF = (5x + y)
(2x – 5)(5x + y)

A polynomial is **<u>non-factorable over the integers</u>** **<u>(Prime)</u>** if it is not possible to factor with only integers as coefficients.

Test for factorability over integers for the quadratic expression $ax^2 + bx + c$

> If $b^2 - 4ac$ is a perfect square, then it can be factored into the product of two binomials $(rx + s)(tx + u)$, where r, s, t, & u are integers(reason why this works in Chapter 8).

This just tells us we can factor, not how to factor.

Example:
$3x^2 + 7x - 27$

> $a = 3$, $b = 7$, $c = -27$: $b^2 - 4ac \rightarrow (7)^2 - 4(3)(-27) \rightarrow 49 + 324 \rightarrow 373$, this is not a perfect square so $3x^2 + 7x - 27$ is not factorable over Integers. We can also say $3x^2 + 7x - 27$ is Prime.

General tips for factoring (other files)

1) Check for common factors, if so factor out the GCF.
   a. If the coefficient of x is negative, factor out -1.
2) Check for the Special Forms (Special Factors)
3) If the coefficient of x is 1 (Trinomial Factoring)
4) If the coefficient of x is not 1
   a. If it is easy to see the factors, use the "Guess & Test" Method (Trinomial Factoring)
   b. If it is not easy to see factors, use the X or AC Method (Factor by X Method)
5) If the polynomial is of higher degree than 2(after GCF), try the Substitution Method (More Factoring)
6) If the polynomial has more than 3 terms, try the Grouping Method (More Factoring)
7) Check for factorability over integers (Non-Factorability)