Factoring Polynomials
More Strategies

To factor by Grouping (4 terms):
    1.)   Group the terms into 2 groups of 2 terms.
    2.)   Factor out the GCF from each group.
    3.)   If there is a common binomial factor, factor it out.
           a. If not, rearrange and try again…

Example 1:
$xy + y + 2x + 2$ GCF1 = y; GCF2 = 2
$y(x + 1) + 2(x + 1)$ Binomial GCF = $(x + 1)$
$(y + 2)(x + 1)$

Example 2:
$16x^3 - 28x^2 + 12x - 21$ GCF1 = $4x^2$; GCF2 = 3
$4x^2(4x - 7) + 3(4x - 7)$ Binomial GCF = $(4x - 7)$
$(4x^2 + 3)(4x - 7)$

Example 3:
$2x^3 - x^2 - 10x + 5$ GCF1 = x^2; GCF2 = - 5 (I know the second one is negative,
    because the second term in the first pair is negative AND the last term is positive)
$x^2(2x - 1) - 5(2x - 1)$ Binomial GCF = $(2x - 1)$
$(x^2 - 5)(2x - 1)$

Example 4:
$5xy - 15x - 6y + 18$ GCF1 = 5x; GCF2 = - 6
$5x(y - 3) - 6(y - 3)$ Binomial GCF = $(y - 3)$
$(5x - 6)(y - 3)$

An expression is **<u>Quadratic in Form</u>** if it is a polynomial of the form $au^2 + bu + c$, where u is any variable term.

Substitution Method for Factoring:
1. Examine the terms to see if you can write them in the above form.
2. Let u = the common expression that makes it Quadratic in Form.
3. Factor as a Quadratic Trinomial.
4. Substitute back in.
5. Check by Multiplying.

Example 1:
$12x^4 + 8x^2 - 7$
$3(2x^2)^2 + 4(2x^2) - 7$ Let $u = 2x^2$
$3u^2 + 4u - 7$ Now Factor
$(3u + 7)(u - 1)$ Substitute $u = 2x^2$ back in.
$(3(2x^2) + 7)((2x^2) - 1)$ Simplify
$(6x^2 + 7)(2x^2 - 1)$ Multiply out to check
$12x^4 - 6x^2 + 14x^2 - 7$
$12x^4 + 8x^2 - 7$

Example 2:
$12x^6 - 32x^3 + 5$
$12(x^3)^2 - 32(x^3) + 5$ Let $u = x^3$
$12u^2 - 32u + 5$ Now Factor
$(6u - 1)(2u - 5)$ Replace u with x^3
$(6x^3 - 1)(2x^3 - 5)$

Factoring higher degree polynomials
1.) If there is a GCF, factor it out. This may reduce the polynomial to a known form.
2.) If it is a Four Term Polynomial try the Grouping Method above.
3.) If it is a Trinomial is it Quadratic in Form?
4.) See if there is an easy factor to find…
5.) Use long division to find what is left
    a. Repeat until you have a quadratic, quadratic form or one of the special forms.
6.) Factor the resultant quadratic if possible.
7.) List all factors found as a product.
8.) Double check by multiplying them all together…