Applications of Linear Equations

Tips for Problem Solving

1) Understand the problem
   a. read & reread the problem(may take several readings), can you restate it in your own words
   b. draw a picture, sometimes it shows better what is going on in the problem
   c. Is there extra or missing information?
2) Devise a plan
   a. List known information
   b. List what is needed to solve the problem
   c. Break the problem into smaller parts, if possible
   d. Translate the problem into an equation
3) Carry out the plan
   a. Work carefully
   b. Keep records so you don't repeat your mistakes
   c. Be patient & keep trying
4) Review your Solutions
   a. Check the solution in the equation(s)
   b. Check the solution in the problem
   c. If it does not work in Both of these, retry
   d. Interpret your results in the context of the problem, if it does not make sense, again retry.

More Tips for Problem Solving

- Draw a picture for triangle or distance problems, if necessary note direction of motion.
- A chart can be helpful for mixture, coin/stamp & some motion problems.
- AC=V, (amount of ingredient) x (cost per unit) = (value of ingredient)
- d=rt, distance = (rate)(time), or r = d/t – as in miles per hour (mph = m/h), Examples 4 & 5 below.
- Q=Ar, Quantity of mixture = Amount(of one substance) x percent of the concentration(this basically a rate in percent converted to decimal form)

	Distance	Rate	Time
With Current			
Against Current			
Total	Total Distance		Total Time

	Quantity	Amount (Pure)	Percent
Solution 1			
Solution 2			
Result	Total Quantity	Total Amount	

- An even integer can be divided by 2, & can be written as 2n.
- An odd integer cannot be divided by 2, & can be written as 2n + 1.
- Consecutive integers follow with no breaks, can be written as n, n+1, n+2…
- Consecutive even (odd) integers can be written as n, n+2, n+4, …; where n is even(odd) (Example 2 below).

We construct models to solve problems: A **model** is a (mathematical) representation of a real world situation (model railroads).

A **formula** is an equation that tells us how to solve the real world situation related to our model. Some formulas model geometric relationships, others model physical relations such as f = ma, force equals mass times acceleration. Having the right formula is sometimes all it takes to solve a problem. Sometimes you can find one already made, d = rt. Sometimes you have to create your own formula; it depends on the problem Examples 1, 2, and 3.

Average

Find the sum of the scores and then divide the sum by the number of scores.

Unit Analysis/Unit Conversion

How many seconds are in 1.2 years?

$$1.2 \text{ years} = ?? \text{ seconds}$$

$$1.2 \text{ years} \cdot \frac{365 \text{ days}}{1 \text{ year}} \cdot \frac{24 \text{ hours}}{1 \text{ day}} \cdot \frac{3600 \text{ seconds}}{1 \text{ hour}} = ?? \text{ seconds}$$

note that the years, days & hours all cancel so only seconds are left on the left

$$1.2 \cdot 365 \cdot 24 \cdot 3600 \text{ seconds} = 37{,}843{,}200 \text{ seconds}$$

$$7672 \text{ seconds} = ?? \text{ hours}$$

$$7672 \text{ seconds} \cdot \frac{1 \text{ minute}}{60 \text{ seconds}} \cdot \frac{1 \text{ hour}}{60 \text{ minutes}} = ?? \text{ hours}$$

$$\frac{7672 \cdot 1 \cdot 1 \text{ hour}}{60 \cdot 60} = 2.13\overline{11} \text{ hours, note the bar above the 11 means the 11 repeats indefinately}$$

2.5 meters are how many feet?

$$2.5 \text{ m} = ?? \text{ ft}$$

$$2.5 \text{ m} \cdot \frac{3.28 \text{ ft}}{1 \text{ m}} = ?? \text{ ft}$$

$$2.5 \cdot 3.28 \text{ ft} = 8.2 \text{ ft}$$

Examples:

1) The larger of two numbers is 4 more than 3 times the smaller. The difference of the two numbers is 16. Find the numbers.

List the Information:

Larger – smaller = 16 **or** Larger – 16 = smaller **or** Smaller + 16 = Larger (the 2nd sentence)

Larger = 4 + 3*smaller (the 1st sentence)

Let x = smaller, so now we have:
x + 16 = Larger & Larger = 4 + 3x → x + 16 = 4 + 3x (This we can solve.)

$x + 16 = 4 + 3x$	
$\underline{-x \qquad\quad -x}$	Subtract x from both sides.
$16 = 4 + 2x$	
$\underline{-4 \quad -4 \qquad}$	Subtract 4 from both sides.
$12 = 2x$	
$\frac{12}{2} = \frac{2x}{2}$	Now Divide both sides by 2.
$6 = x$	Remember x is the smaller number, we still need the larger number!

x + 16 = Larger	Substitute x = 6.
(6) + 16 = Larger	
22 = Larger	

So the two numbers are 22 and 6.

2) Seven times the sum of three consecutive odd integers is 24 less than three times the largest. Find the numbers.

List the information:

3 consecutive odd integers are: x, x + 2, x + 4.

7[x + (x + 2) + (x + 4)] = 3(x + 4) - 24 (This we can solve.)

7[x + (x + 2) + (x + 4)] = 3(x + 4) - 24	1st simplify each side of the equation,
7*x + 7*(x + 2) + 7*(x + 4) = 3*x + 3*4 - 24	using the distributive property.
7x + 7x + 14 + 7x + 28 = 3x + 12 - 24	2nd combine like terms.
21x + 42 = 3x + -12	Now move “x’s” to left & numbers to right.
$\underline{-3x \; -42 \quad -3x \; -42}$	
18x = - 54	Divide both sides by 18
x = - 3	This is just x! Need also x + 2 & x + 4!

x + 2 → (-3) + 2 → -1
x + 4 → (-3) + 4 → 1
So the three consecutive odd integers are:
-3, -1, 1

3) Bud can paint a house in 40 hours, while Sean can paint the same house in 50 hours. How long will it take them to paint it if they work together?

Some things to keep in mind:

If they work together they will take less time than the faster person.
Each person will work a fraction of the whole time.
The rate at which each person works is 1 hours/# of hours.
rate * time = work done
Let t = time

The portion painted by Bud + The portion painted by Sean = The whole house

$$\frac{1}{40}t + \frac{1}{50}t = 1 \rightarrow \frac{t}{40} + \frac{t}{50} = 1 \xrightarrow{LCD=200} 200\left(\frac{t}{40} + \frac{t}{50}\right) = 200 * 1 \xrightarrow{Distribute}$$

$$5t + 4t = 200 \rightarrow 9t = 200 \xrightarrow{\text{Divide both sides by 9}} t = \frac{200}{9}$$

(Don't forget to put in units. It is a good idea at this point to write a complete sentence!)

Together Bud and Sean will paint the house in $22\frac{2}{9}$ hours.

4) Two trains travel in opposite directions, left Denver at the same time. The little blue train chugged along at 54 miles an hours, and the smaller green train puffed along at 49 mph. How long will it be for them to be 180 miles apart?

What we know:

d = rt
The total distance is 180 miles.
Let t = time

	Distance	Rate	Time
Blue Train	54t	54	t
Green Train	49t	49	t
Total	180		

So we have for our equation:

54t + 49t = 180 Simplify the left.
103t = 180 Divide both sides by 103.

$$t = \frac{180}{103} \text{ or } t \approx 1.75$$

The will be 180 miles apart in about 1.75 hours.

5) It takes an airplane 15 minutes longer to complete a flight between two cities when it has a 50 mph headwind than it takes when the wind is not blowing. The plane normally cruises at 500 mph when the wind is not blowing. (a) How long does the trip take when the plane has the headwind? (b) How far apart are the cities?

Information that we have:

Speed = 500 mph with no wind.
Time with 50 mph head wind = Time without headwind PLUS 15 minutes
15 minutes = .25 hours
d = rt
headwind slows the planes speed by 50, so speed with the headwind it 450 mph.
Let t = time without headwind

	Distance	Rate	Time
Without headwind	500t	500	t
Against headwind	450(t + .25)	450	t + .25

Since the plane is traveling the same distance in each case we have:
Speed with headwind * (time + .25 hours) = speed without headwind * time
OR
450(t + .25) = 500t (This we can solve)

$$450(t + .25) = 500t$$ Distribute the 450 on the left.

$$450t + 112.5 = 500t$$

$$\underline{-450t \qquad\qquad -450t}$$ Subtract 450t from each side.

$$112.5 = 50t$$

$$2.25 = t$$ Divide both sides by 50

The time is 2.25 hours, when the wind is not blowing! So it takes 2.25 hours PLUS .25 hours or 2.5 hours when the wind is blowing.(a)
Distance = rate * time, so we have for part (b)
d = 500(2.25)
d = 1125
The distance is 1125 miles. (b)
(It is sometimes nice to write all the answers together.)

The plane takes 2.5 hours with a 50mph headwind to travel the 1125 miles between the two cities.