Average, Weighted Average and Mixture Problems

To find the **average** of a set of *n* numbers, add the numbers in the set and divide the sum by *n*. One use is for the grade for a course.

Example:

Fred has test grades: 88, 45, 96, 72 and he still must take one more test. What does he need in order to get a 70 for the course?

What we know:

The average is the sum of the numbers divided by the number of numbers(5).

Let x = the missing grade.

$$\frac{88+45+96+72+x}{5}=70\xrightarrow{Multiply\_both\_sides\_by\_5}5*\left(\frac{88+45+96+72+x}{5}\right)=5*70$$

$$\rightarrow 88+45+96+72+x=350\xrightarrow{Simplify+Left\_Side}301+x=350\xrightarrow{Subtract\_301\_from\_both\_sides}x=49$$

Fred needs a 49 on the test to get a 70 in the course.

The **weighted average** of a set of numbers is a special kind of average, in which some elements of the set carry more importance (weight) than others. One use is for GPA. Course grade is another use, for example homework may be 25% of the grade, quizzes 25% of the grade and exams 50%.

Example:

Course(optional)	Letter Grade(optional)	Value of Grade	Credit Hours	Value * Credit Hours
Computers & Problem Solving	B+	3.5	3.0	3.5*3 = 10.5
Calculus II	B	3.0	4.0	12
PE	C+	2.5	1.0	2.5
Religions of the East	A	4.0	3.0	12
Russian II	B	3.0	3.0	9
Total			14.0	46

The credit hours in this problem are the weight. The 4 credit course is worth more to GPA than any of the others. The 1 credit PE course hardly affects the GPA. The GPA for this semester is (Total of Value * Credit Hours)/(Total Credit Hours) → 46/14 → 3.286.

Mixture Problems are like the weighted average, the “weight” of the elements varies with the problem. The “weight” can be cost, percent of a solution, and many other types of units.

Examples:

1) A car’s radiator has 24 quarts of a 40% antifreeze solution. How many quarts should be drained and replaced with pure antifreeze if the final mixture is to be 50% antifreeze?

What we know:

Total antifreeze is 24 quarts.
Pure antifreeze is a 100% solution or 1.0
40% solution = .4
Let x = the amount of 40% we take out of the radiator = the amount of 100% we put into the radiator.

	Amount of Mixture	Percent Concentration	Amount (Pure)
40% Solution	24 - n	.40	.4*(24 – n)
100% Solution	n	1.0	1*n
Result 50% Solution	24	.5	.5*24

The amount of pure antifreeze in the result is the sum of what is put into the radiator so:

$.4(24 - n) + n = .5*24$	Simplify each side
$9.6 - .4n + n = 12$	
$9.6 + .6n = 12$	We could multiply the entire equation by 10 to
$\underline{-9.6 \qquad\qquad -9.6}$	remove decimals, but I don’t think it helps, so I
$.6n = 2.4$	will subtract 9.6 from both sides.
$n = 4$	Divide both sides by .6

Four quarts of the 40% solution needs to be removed so 4 quarts of the 100% solution can be put it to increase the percentage to 50%.

2) Strasburg Railroad charges 2 fares for a Coach pass, $7.50 for children and $15 for adults. If the total is $5077.50 for 423 tickets, how many of each type were sold?

Let t = the number of children's tickets sold.

	Number Tickets	Cost	Revenue
Child	t	7.5	$7.5t$
Adult	$423 - t$	15	$15(423 - t)$
Result	423		5077.5

$7.5t + 15(423 - t) = 5077.5$

$7.5t + 6345 - 15t = 5077.5$

$6345 - 7.5t = 5077.5$ Subtract 6345 from each side.

$\underline{-6345 \qquad\quad -6345}$

$-7.5t = -1267.5$

$t = 169$ Divide both sides by -7.5

The number of children's tickets were 169, and adult tickets were 423 – (169) = 254.