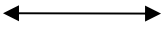
Slope

The **slope** (m) of a line describes the slant of the line. It is also the **average rate of change in y over x**. If $P_1(x_1, y_1)$ & $P_2(x_2, y_2)$ are 2 points on the line then $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{change in y}}{\text{change in x}} = \frac{rise}{run}$, so long as $x_2 \neq x_1$.

The slope of any horizontal line is 0.

The slope of any vertical line is undefined.

If m < 0 

If m = 0 

If m > 0 

If m is undefined 

To find the slope of a line determined by the 2 points (x_1, y_1) and (x_2, y_2) use the slope formula given above.

Example: Find the slope of the line determined by the points (4, -2) and (-5, 3). Note: The point that you choose to be point 1 does not matter.

First, I will choose (4, -2) to be point 1, and (-5, 3) to be point 2.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - (-2)}{(-5) - (4)} = \frac{3 + 2}{-5 - 4} = \frac{5}{-9} = -\frac{5}{9}$$

Now, I will choose (-5, 3) to be point 1, and (4, -2) to be point 2.

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-2) - (3)}{(4) - (-5)} = \frac{-2 - 3}{4 + 5} = \frac{-5}{9} = -\frac{5}{9}$. So either way the slope of the line is $-\frac{5}{9}$.

Example: Sign on the side of the road warning truck drivers that the hill has a 10% grade. Since percents can be written as fractions this means for every 10 feet down there is 100 feet horizontal.

Example: Roof slope is determined by vertical rise over horizontal run, expressed in inches, the second number always being 12. For example, if a roof rises at a rate of 6 inches per foot (horizontal), it is a 6/12-slope roof. Keep in mind in Math we would reduce $\frac{6}{12}$ to $\frac{1}{2}$. From: http://www.usinspect.com/Roof/roof.asp

Parallel and Perpendicular Lines

Two non-vertical lines are **<u>parallel</u>** if and only if the slopes of the two lines are equal (the lines never intersect). Any two *vertical lines* are parallel. Any two *horizontal lines* are also parallel since all *horizontal lines* have slope = 0.

If m_1 & m_2 are the slopes of 2 lines (neither vertical), then the lines are **<u>perpendicular</u>** if and only if $m_1 = -\frac{1}{m_2}$. Another way to check if two lines are perpendicular is if the product of their slopes is -1, $m_1 m_2 = -1$. A vertical line is perpendicular to a horizontal line. The x-axis is perpendicular to the y-axis.

Examples:

1) Determine if Lines 1 and 2 are parallel, perpendicular or neither. Line 1 has slope m = 3 and is through the point (0, 5), and Line 2 is through the points (0, 8) and (1, 11).
    a. To determine if the lines are parallel, perpendicular or neither, we just need to compare the slopes of the lines.
    b. The Slope of Line 1 is given as m = 3.
    c. Line 2, we are given two points on the line, so we need to calculate the slope. $m = \frac{y_2 - y_1}{x_2 - x_1} \rightarrow m = \frac{11 - 8}{1 - 0} \rightarrow m = \frac{3}{1} \rightarrow m = 3$
    d. Slope of Line 1 = 3 = Slope of Line 2
    e. Since the slopes of the two lines are the same the lines are parallel.

2) Determine if Lines 1 and 2 are parallel, perpendicular or neither. Line 1 has slope m = 3 and is through the point (0, 5), and Line 2 is through the points (0, 8) and (3, 7), then these two lines are perpendicular.
    a. To determine if the lines are parallel, perpendicular or neither, we just need to compare the slopes of the lines.
    b. The Slope of Line 1 is given as m = 3.
    c. Line 2, we are given two points on the line, so we need to calculate the slope. $m = \frac{y_2 - y_1}{x_2 - x_1} \rightarrow m = \frac{7 - 8}{3 - 0} \rightarrow m = \frac{-1}{3} \rightarrow m = -\frac{1}{3}$
    d. (Slope of Line 1) * (Slope of Line 2) → $3 * \left(-\frac{1}{3}\right) \rightarrow -1$
    e. Since the product of the slopes is -1, the two lines are perpendicular.